



# Particle Acceleration at Astrophysical Shocks

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# Outline of Lectures

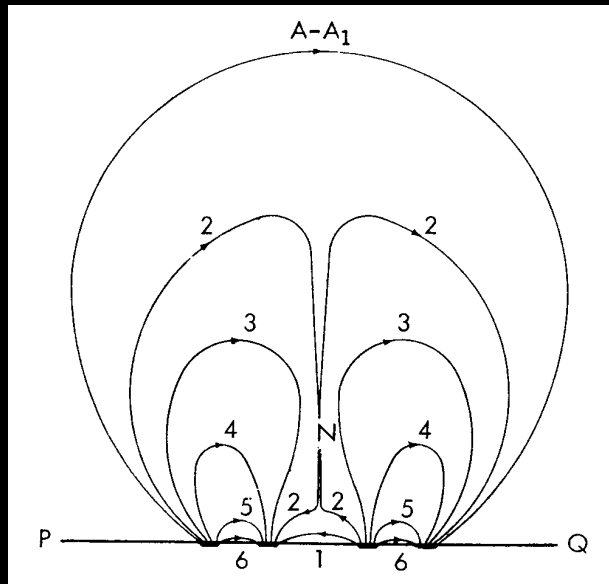
- 1. Astrophysical source contexts;
- 2. Cosmic ray acceleration: Fermi's original idea;
- 3. Non-relativistic, test-particle shocks: canonical power-law generation and efficiency issues;
- 4. Genres of theoretical approaches;
- 5. Non-linear dynamical effects in strong shocks: cosmic ray modification;
- 6. Nuances: magnetic field amplification;
- 7. Relativistic shocks: non-canonical power-laws, acceleration times and thermalization vs. acceleration.

Red denotes today's topics

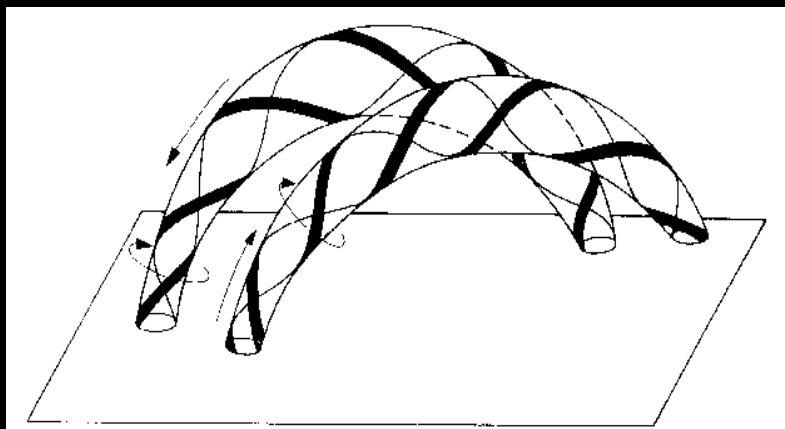
# Ubiquity of Shock Acceleration

- Signatures of *collisionless shock acceleration* are seen everywhere in the universe: supersonic flows abound;
- Dissipation in such tenuous systems as *not at maximal entropy*: opens up the possibility of **non-thermal acceleration / deceleration**, **electrodynamic energization**, and **broad inertial ranges for magnetic turbulence**;
- Accelerated particles form an integral part of the shock structure in addition to magnetohydrodynamics (MHD);
- Other types of cosmic particle acceleration include:
  - **Stochastic energization** (e.g. Fermi's original idea) in turbulent systems: solar flares, accretion flows;
  - **Magnetic reconnection** at X-points in dynamic, large-scale fields: solar flares, solar and pulsar winds;
  - **Coherent electric fields**: pulsars.

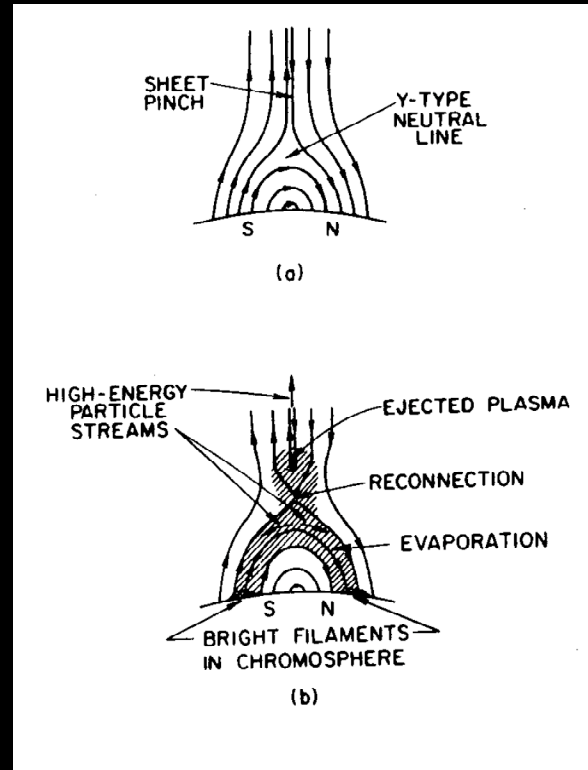
# Magnetic Reconnection Scenarios



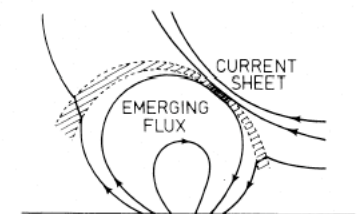
Sweet, (1958)



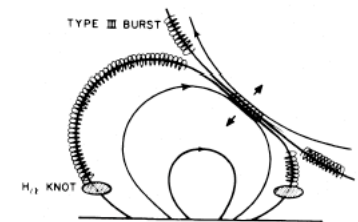
Gold and Hoyle, (1961)



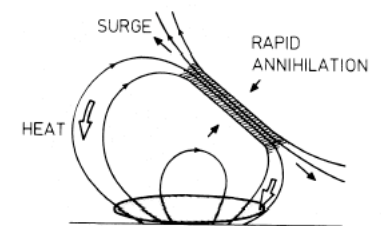
Sturrock, (1966)



(a) Preflare Heating



(b) Impulsive Phase



(c) Main Phase

Heyvaerts, Priest and Rust, (1977)

See <http://solarmuri.ssl.berkeley.edu/~hudson/cartoons/overview.html>

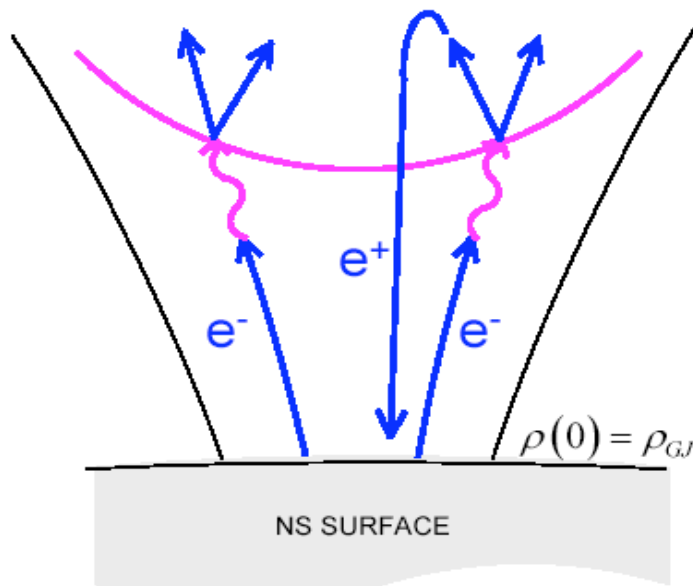
# Electric Field Acceleration in Pulsars

## *Polar cap accelerators*

**SPACE CHARGE "GAP"**

$$T_s > T_{e,i}$$

$$\Omega \cdot B > 0$$

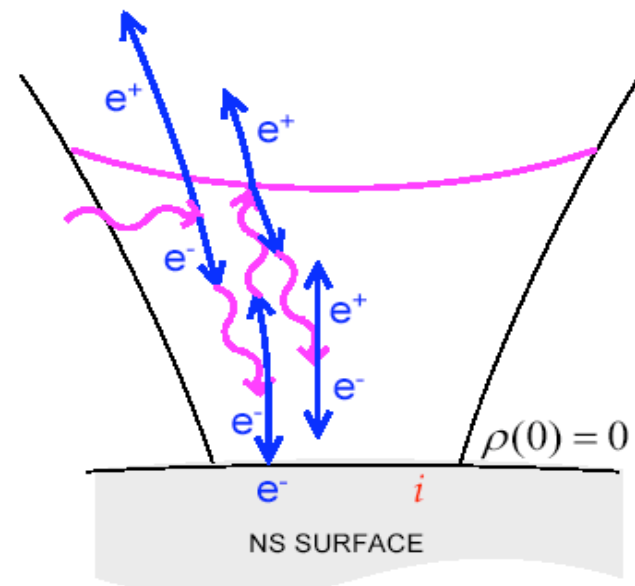


$$\nabla \cdot E = -4\pi(\rho - \rho_{GJ})$$

**VACUUM GAP**

$$T_s < T_{e,i}$$

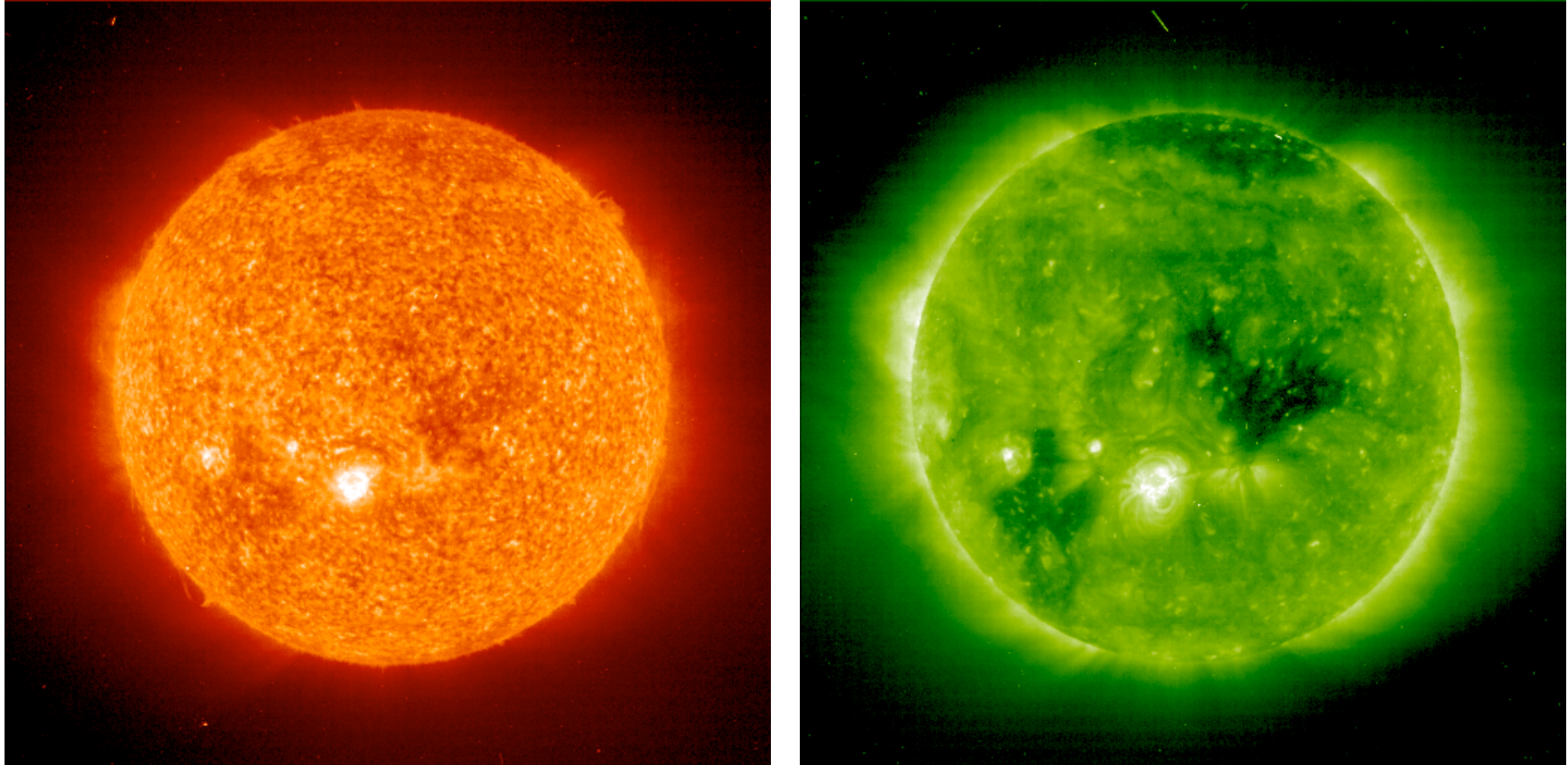
$$\Omega \cdot B < 0$$



$$\nabla \cdot E = 4\pi\rho_{GJ}$$

Courtesy: Alice Harding

# Sun in UV: SOHO Observations

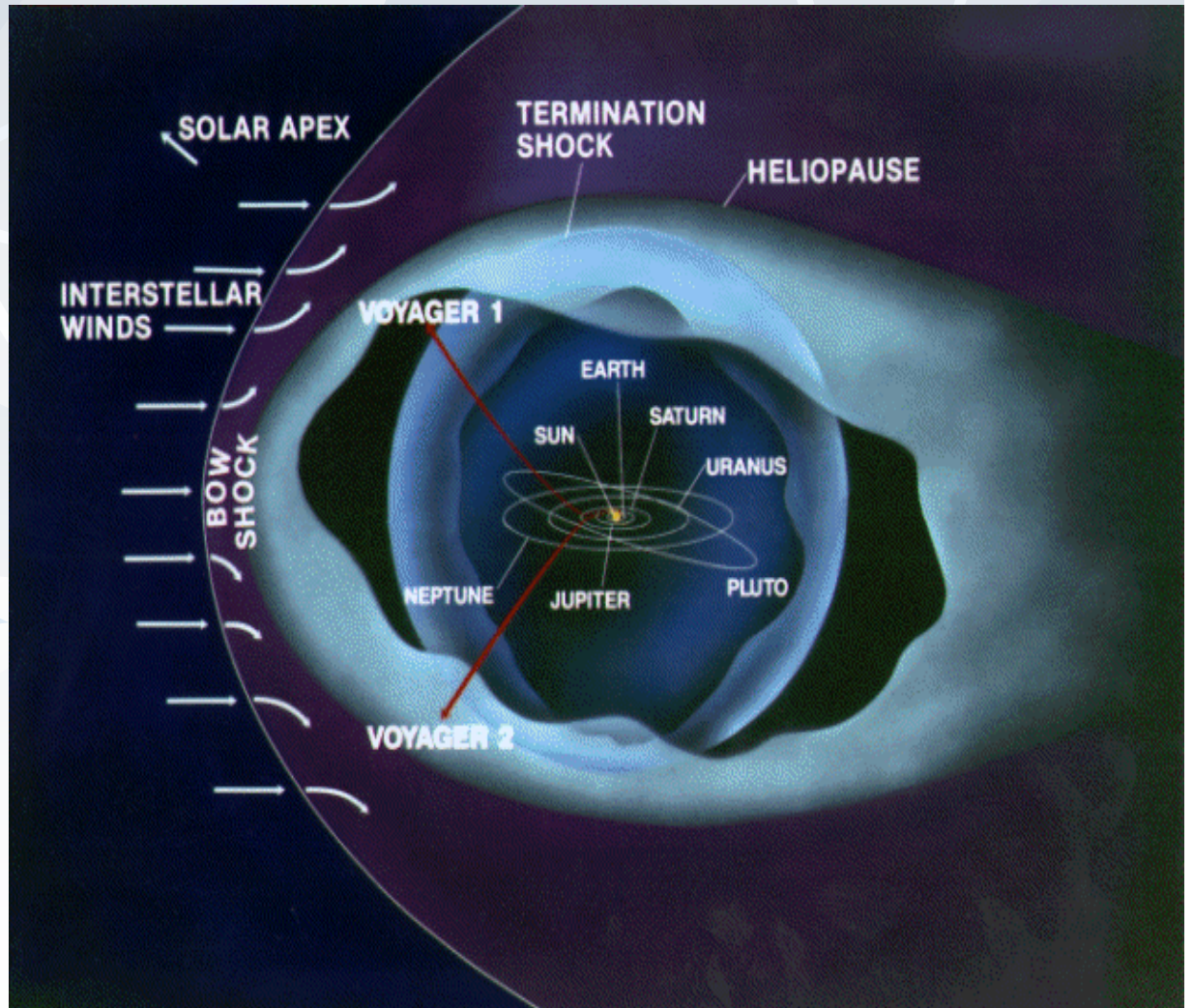


- Left panel: He lines showing convective granulation
- Right panel: Fe emission exhibiting coronal activity



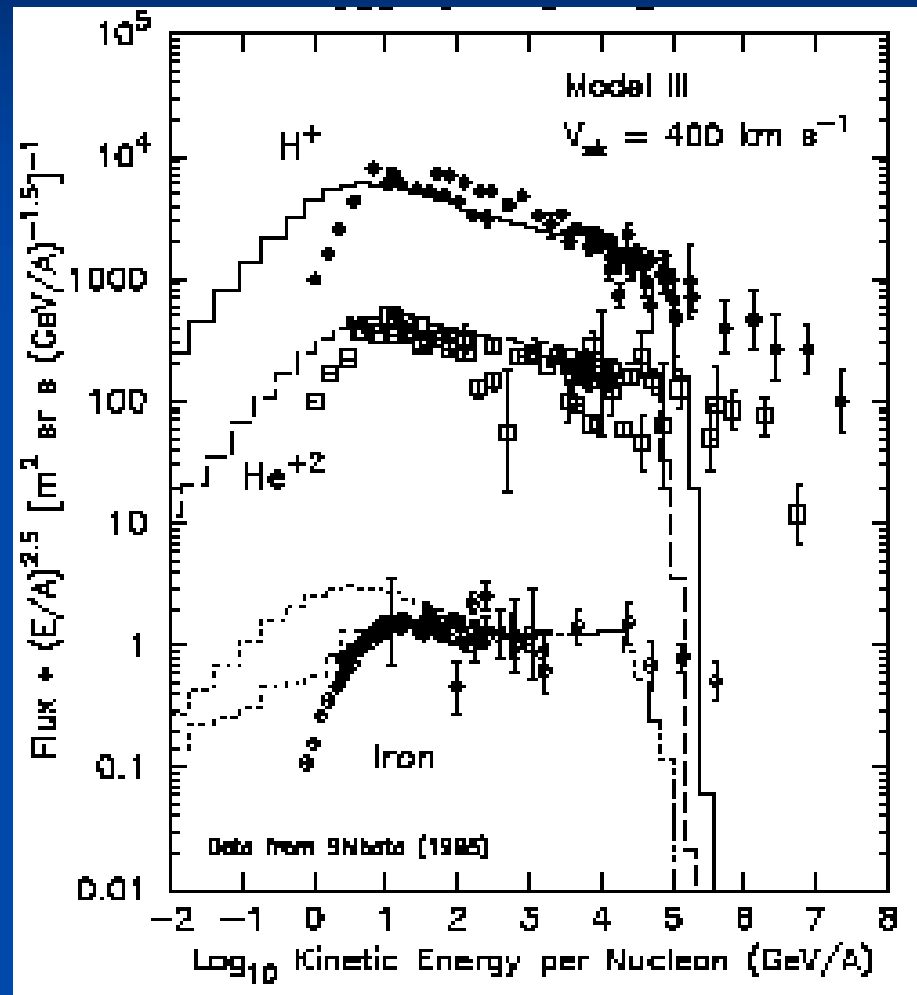
# Shocks in the Heliosphere

- *Planetary bow shocks:* usually strong, with nonlinear acceleration being important.
- *Interplanetary travelling shocks:* usually low Mach number, with a big contribution from **interstellar pick-up ions**;
- *Solar wind termination shock:* site of **anomalous cosmic ray** generation [Voyager I was there, 2005?].



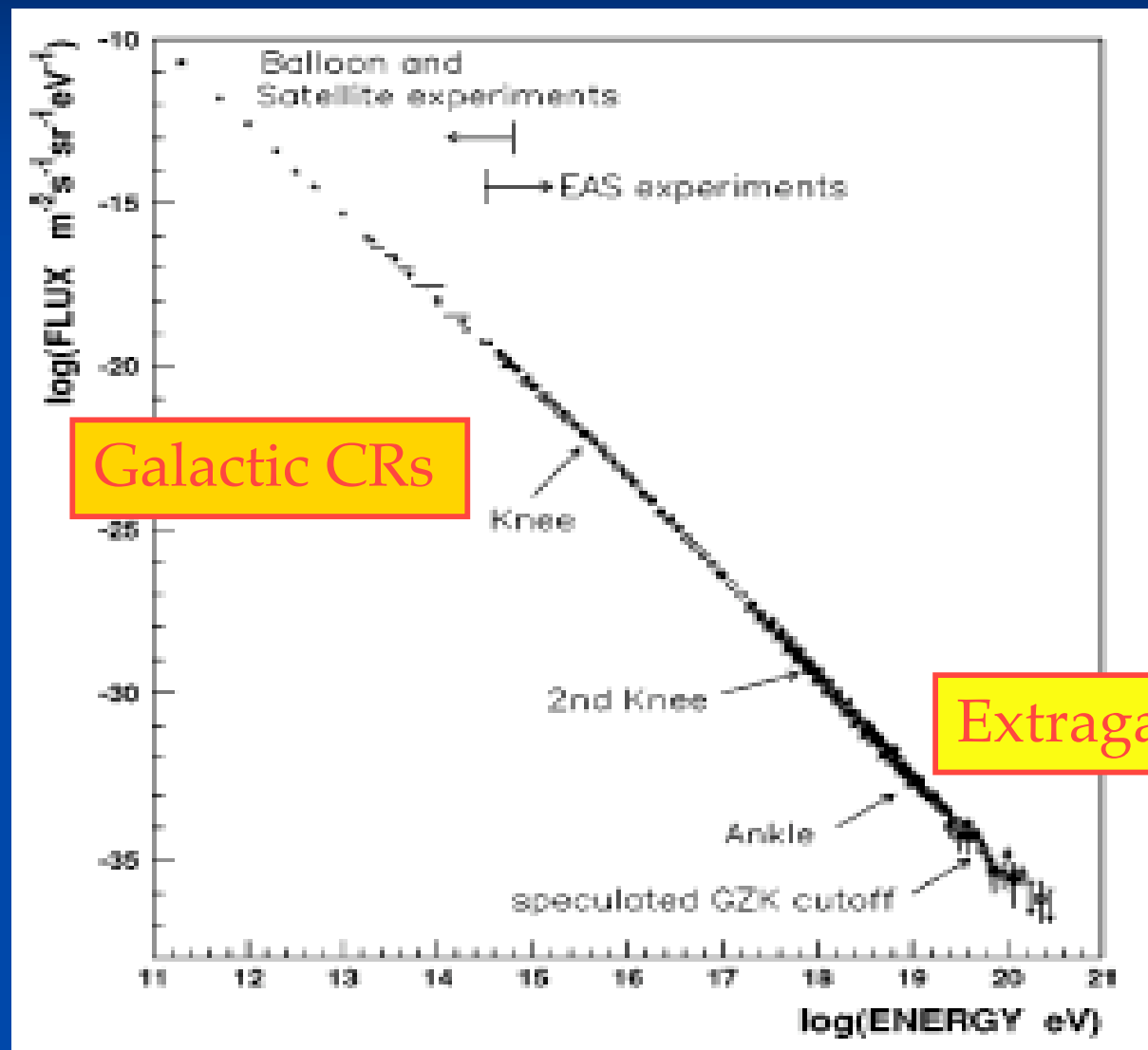
# Galactic Cosmic Rays

- SNR origin?
- Solar modulation reduces flux below 1 GeV/nucleon;
- Instrumental data spread increases near CR knee;
- Non-linear models of acceleration required for SNRs: **abundances are not solar**;
- [Ellison et al. 1997]

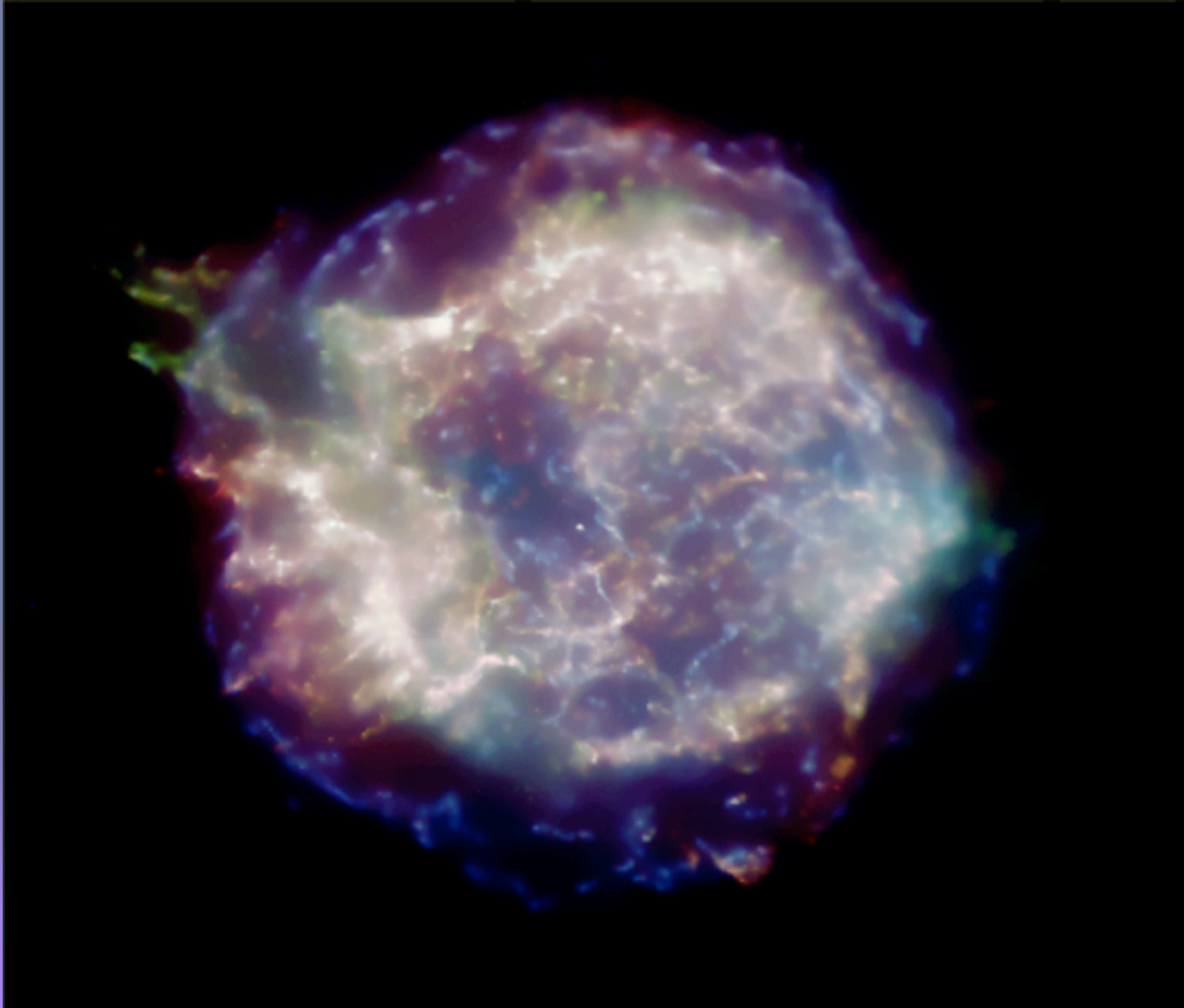




# Complete Cosmic Ray Spectrum



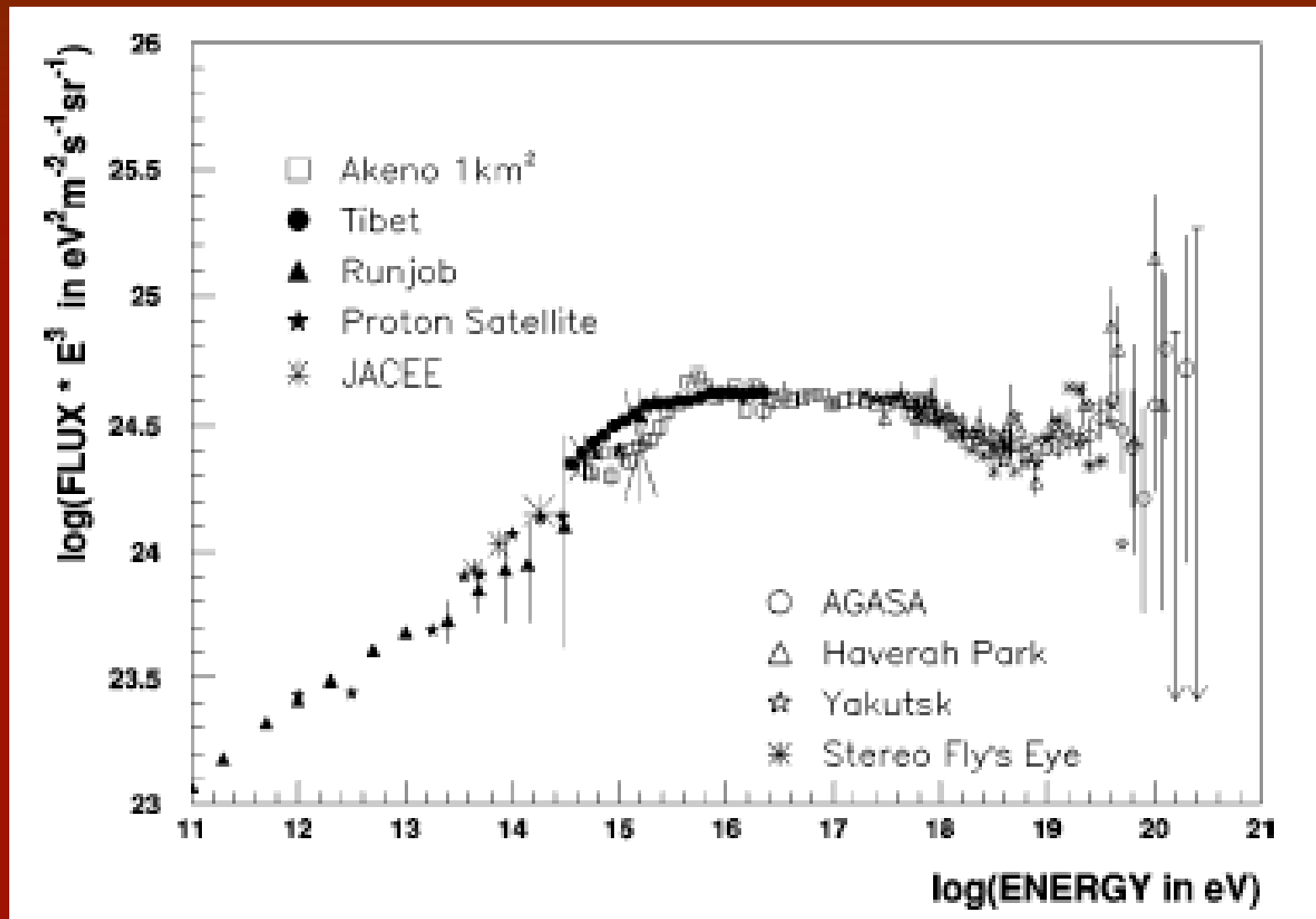
# Cassiopeia A Supernova Remnant



# Crab Nebula: X-ray/Optical Montage

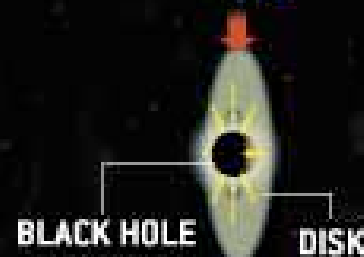


# High Energy Cosmic Ray Spectrum



## BURSTING OUT

### MERGER SCENARIO



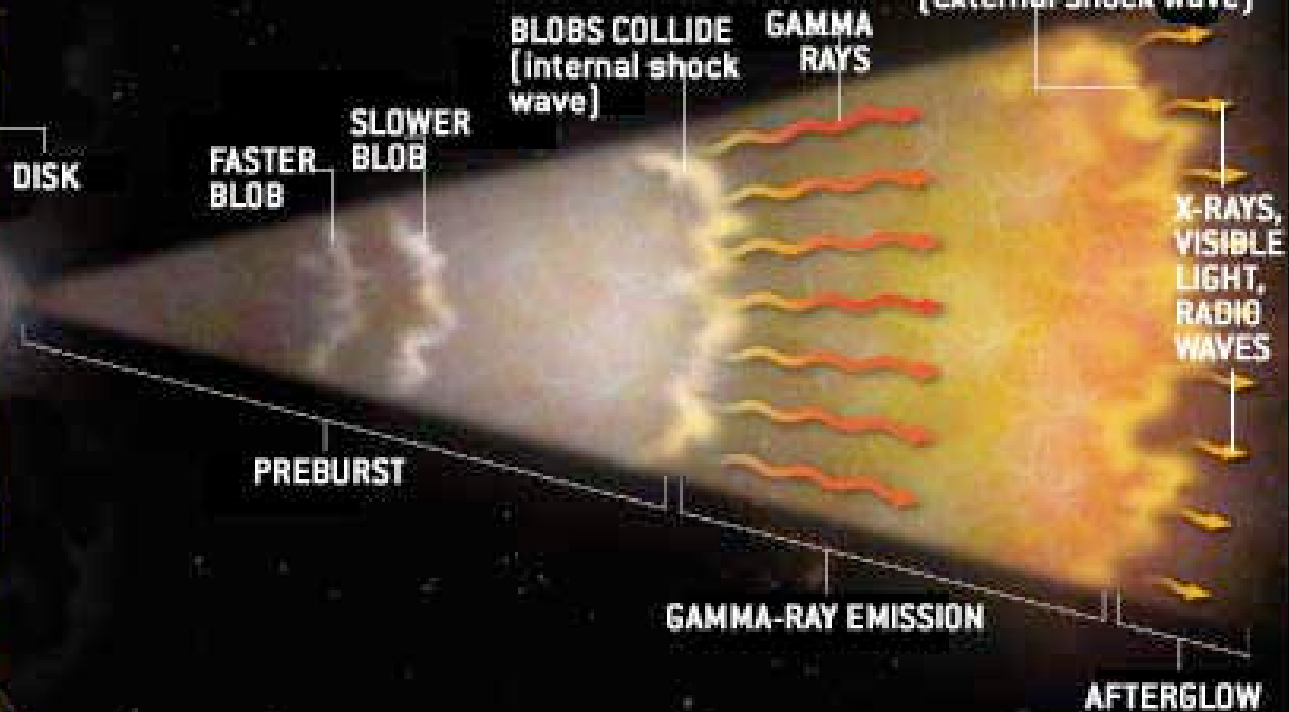
CENTRAL  
ENGINE



### HYPERNOVA SCENARIO

JUAN VELASCO

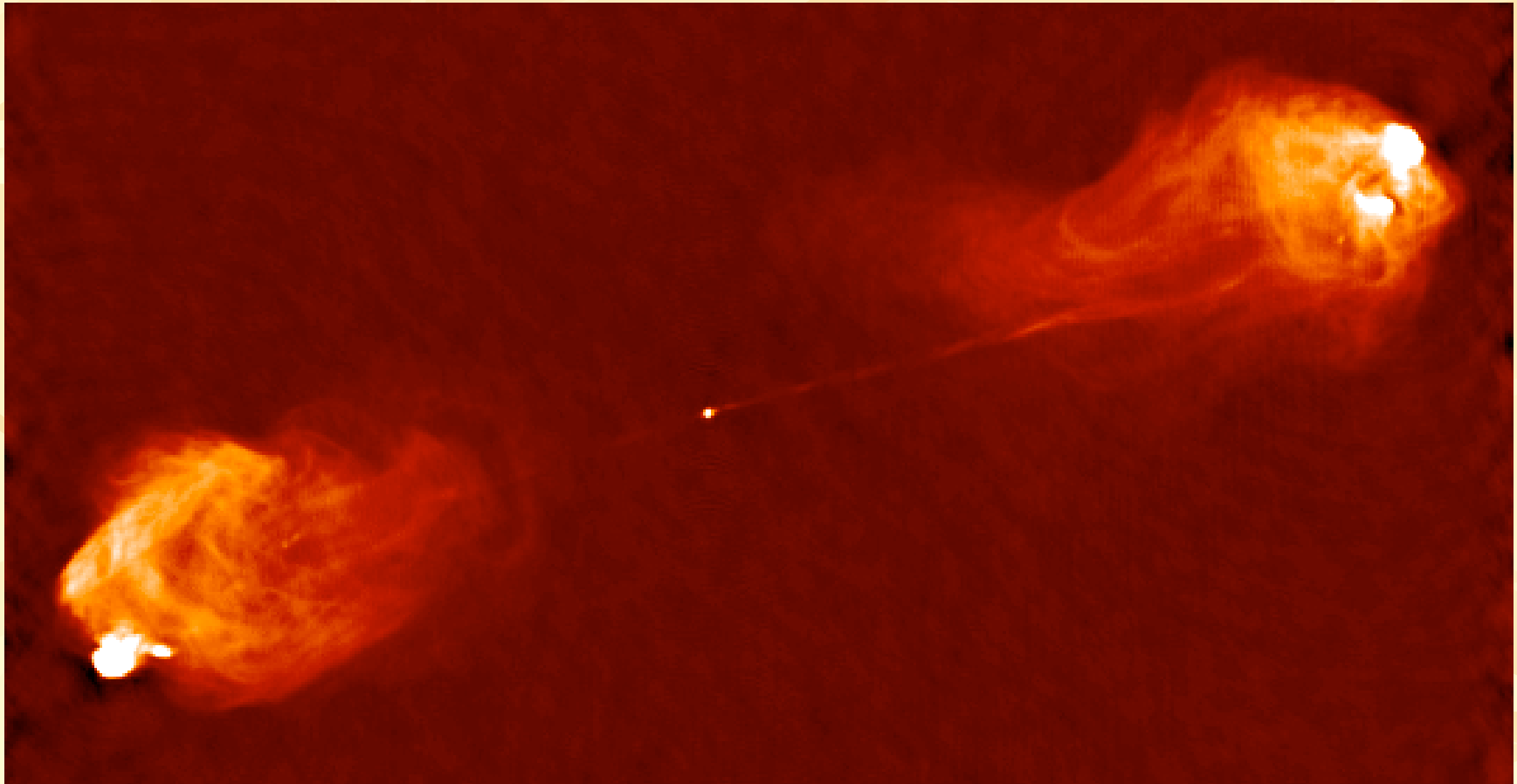
FORMATION OF A GAMMA-RAY BURST could begin either with the merger of two neutron stars or with the collapse of a massive star. Both these events create a black hole with a disk of material around it. The hole-disk system, in turn, pumps out a jet of material at close to the speed of light. Shock waves within this material give off radiation.



Courtesy of *Scientific American*



# High Energy Cosmic Ray Accelerators: Radio Galaxies like Cygnus A



# Rogue's Gallery of Cosmic Acceleration Sites

- Stellar coronae (Guedel; week 1)
- Supernova remnants (Vink; week 2)
- Pulsar wind nebulae (Grenier; week 2)
- Pulsars (Hermsen; week 2)
- Galactic X-ray binaries (Belloni; week 2)
- Sgr A\* (Eckart; week 2)
- Galaxy clusters (Arnaud; week 1)
- Relativistic jets (Corbel; weeks 1+2)
- Gamma-ray bursts (Daigne; week 1)
- Active galactic nuclei (Fabian; week 2)

# Fermi's Original idea

PHYSICAL REVIEW

VOLUME 75, NUMBER 8

APRIL 15, 1949

## On the Origin of the Cosmic Radiation

ENRICO FERMI

*Institute for Nuclear Studies, University of Chicago, Chicago, Illinois*

(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

- Application: cosmic ray production in the interstellar medium between colliding gas clouds;
- **Stochastic acceleration concept**, now referred to as 2nd order Fermi process.
- May or may not be dominant acceleration mechanism in an astrophysical shock.

# Diffusive/Stochastic Acceleration

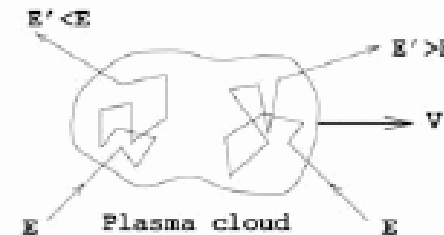
- **1st order Fermi** uses kinematic gains in shock crossings to accelerate particles;
- **2nd order Fermi** uses bias towards head-on collisions to effect acceleration.
- Respectively described by **friction** and **diffusion** terms in Fokker-Planck equation for momentum changes.

Graphic: courtesy HIREs collaboration

## Fermi Acceleration Mechanism

Stochastic energy gain in collisions with plasma clouds

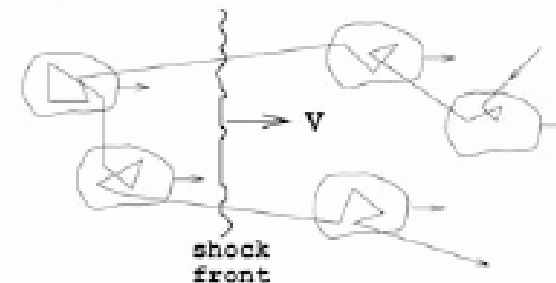
2nd order :  
randomly distributed magnetic mirrors



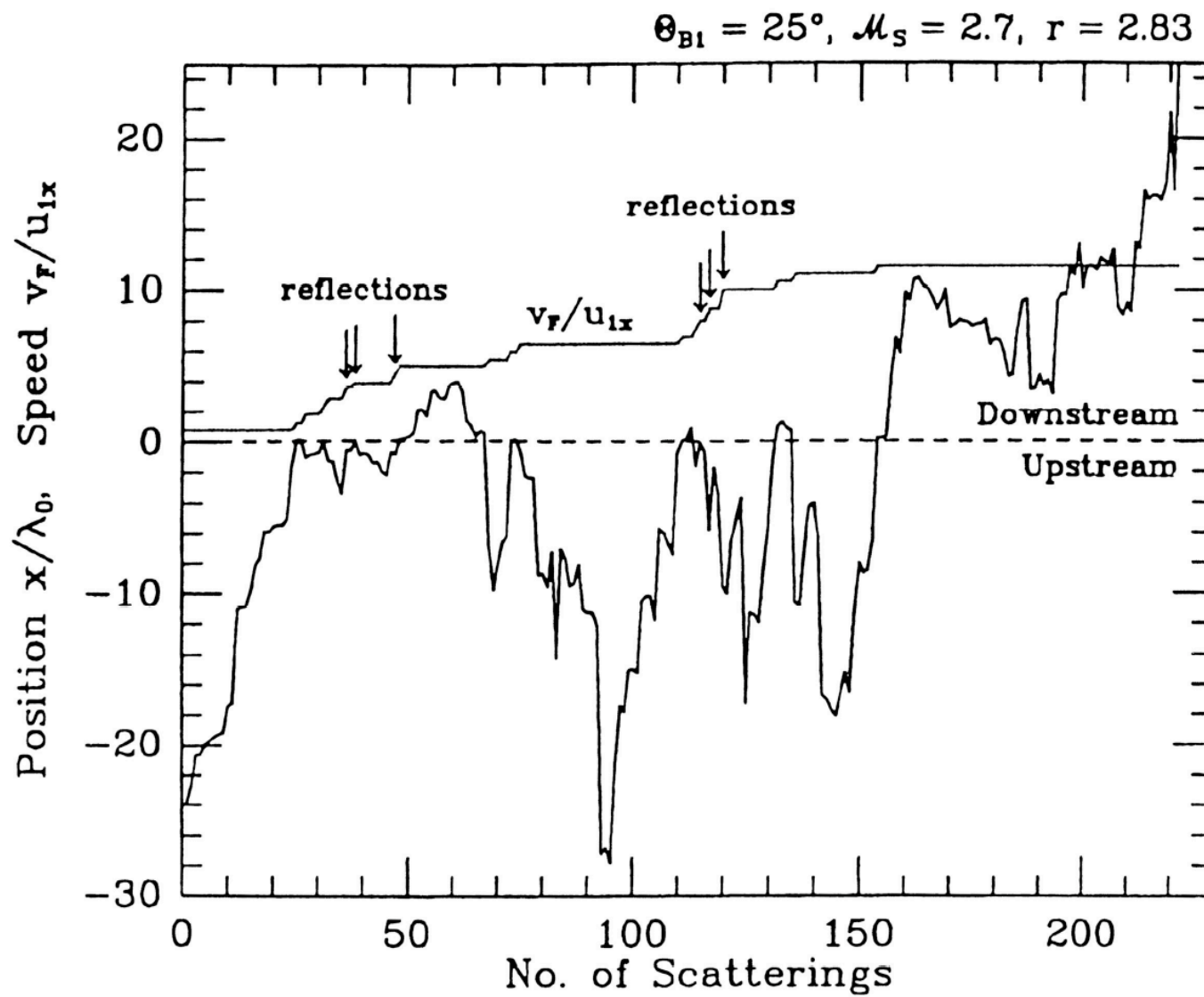
$$\frac{\Delta E}{E} \sim \beta^2 \quad \beta = \frac{V}{c} \lesssim 10^{-4}$$

[Slow and inefficient]

1st order :  
acceleration in strong shock waves  
(supernova ejecta, RG hot spots...)

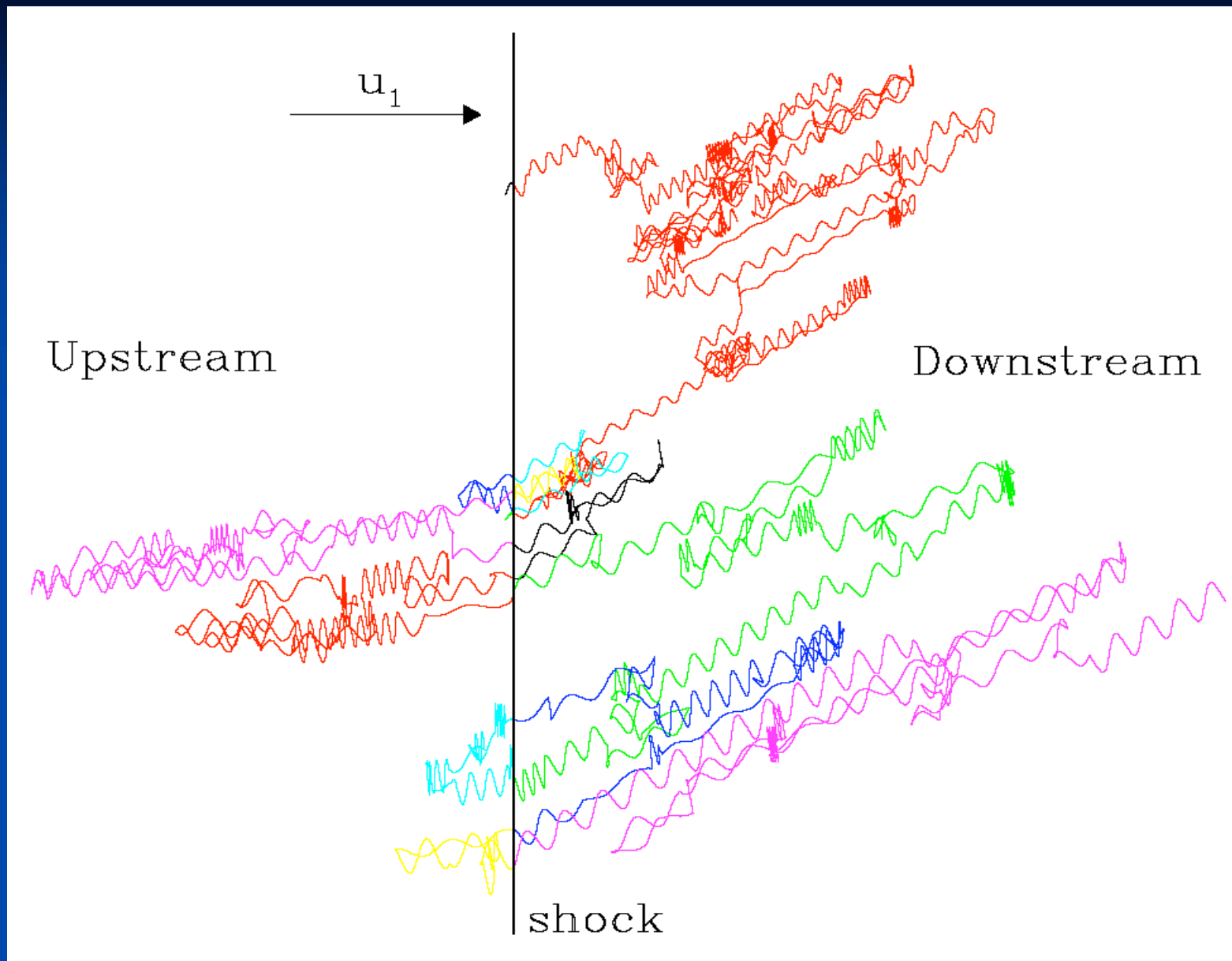


$$\frac{\Delta E}{E} \sim \beta \quad \beta = \frac{V}{c} \lesssim 10^{-1}$$



Baring, Ellison & Jones (1994)

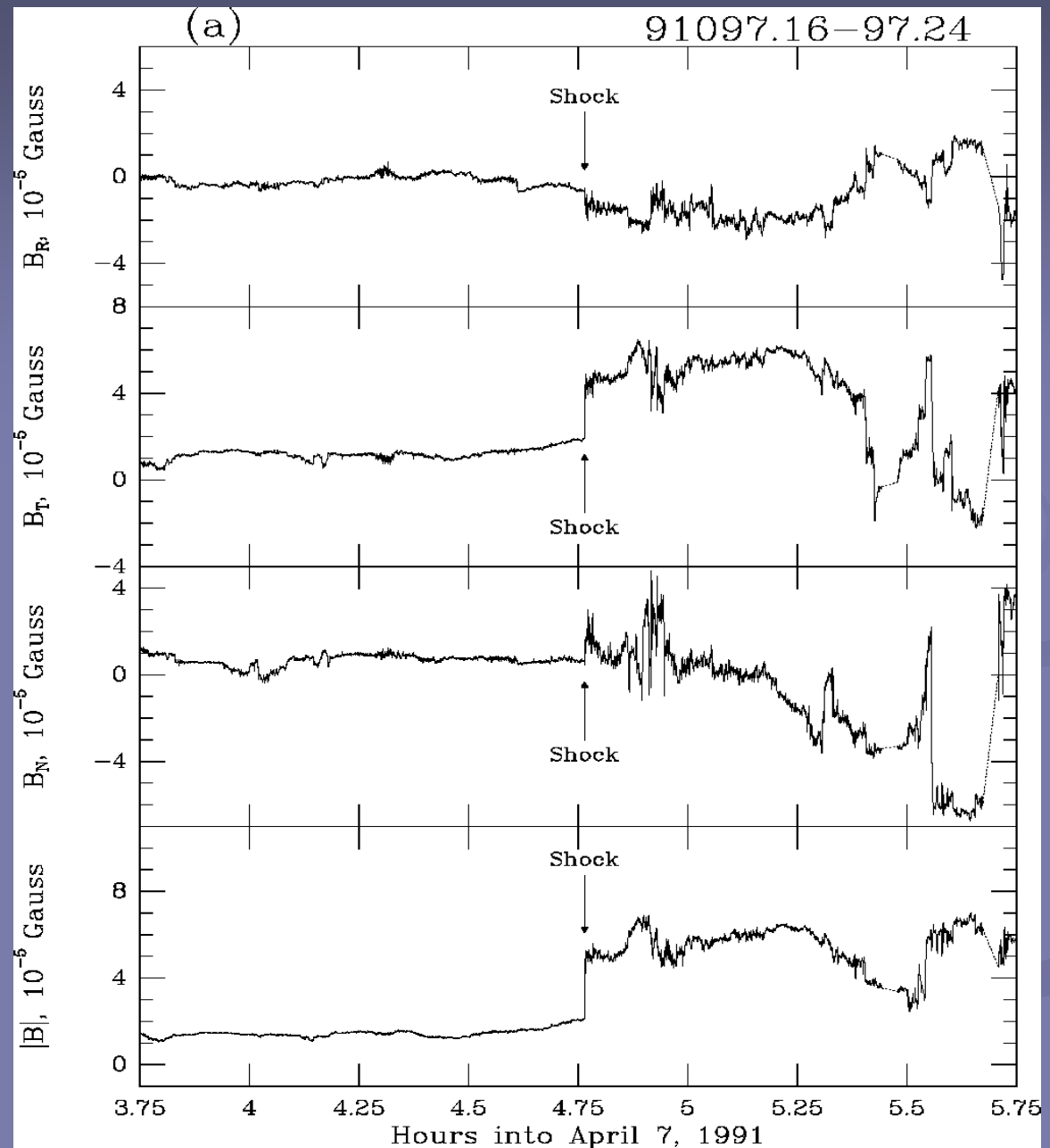




Baring & Summerlin (2006)

# Ulysses Shock 91097

- Interplanetary shock turbulence
- RTN field data from Ulysses' magnetometer;
- Turbulent enhancement at shock;
- Turbulence strong enough to generate strong cross field diffusion.



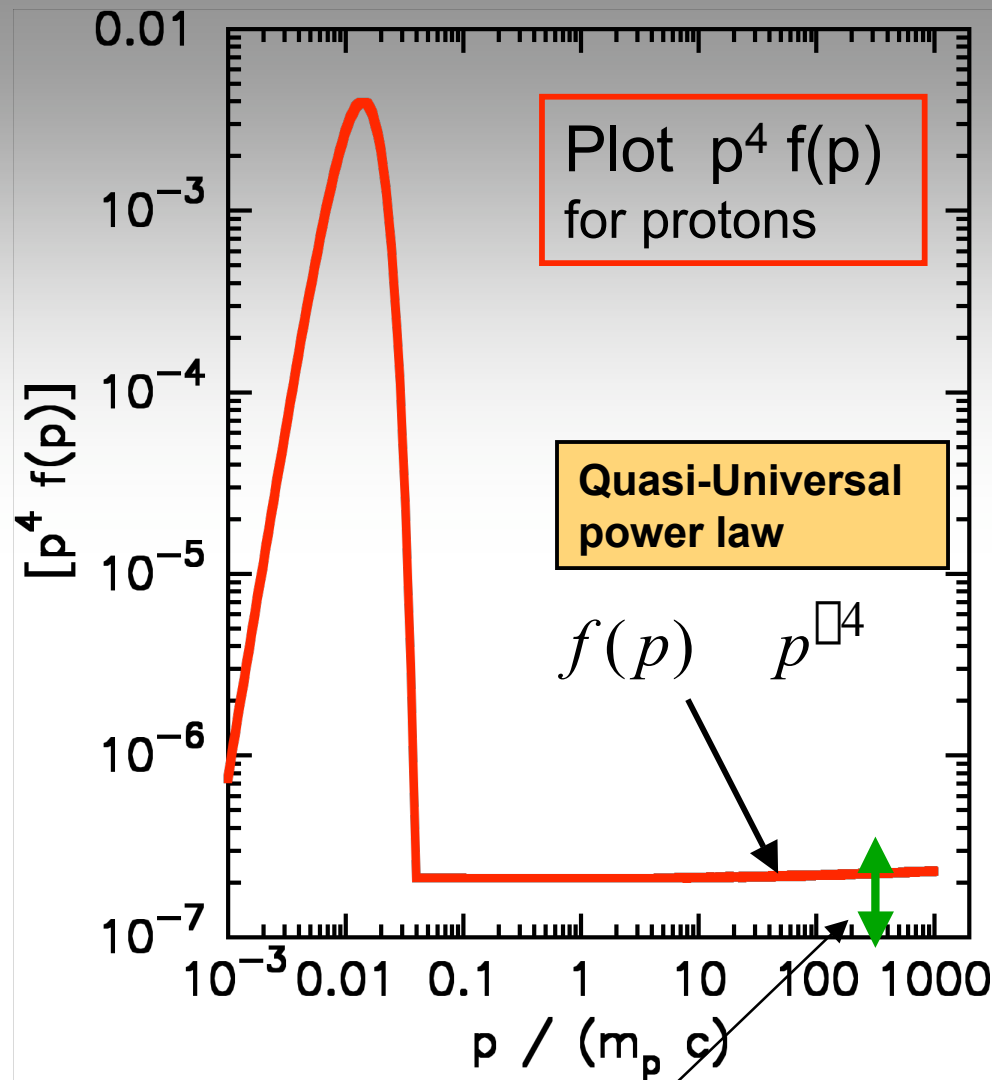
Baring et al. (1997)



# Properties of Non-relativistic shocks

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- **Isotropy** of high energy particles in all pertinent frames guarantees a power-law with a canonical index (*diffusion approximation*);
- Normalization of power-law anti-correlates with shock obliquity, but **index is independent of obliquity**;
- Diffusion across field lines facilitates efficient injection in oblique shocks;
- Non-linear feedback between acceleration and hydrodynamics yields spectral curvature.



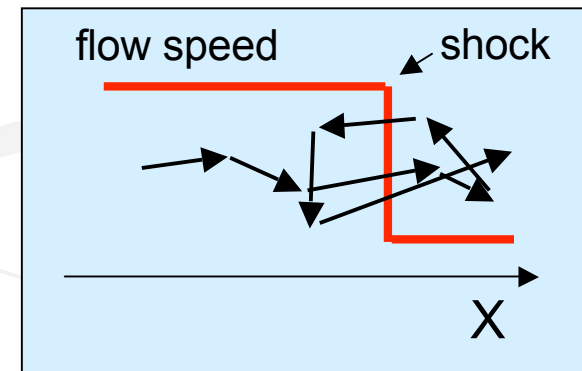
Normalization of power law not defined in test-particle approximation.

## Test particle power-law in diffusive shock accel.

Krymsky 77, Axford et al 77, Bell 78, Blandford & Ostriker 78

$f(p) \sim p^{-3r/(r-1)}$  where  $r$  is compression ratio,  $f(p)$  is phase space density

If  $r = 4$ , &  $\gamma = 5/3$ ,  
 $f(p) \sim p^{-4}$



Test particle results: **ONLY** for superthermal particles, no information on thermal particles, or their injection.



# Power-Law Index from Diffusive Shock Acceleration

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- In a steady-state scenario, the integral distribution function is governed by the following kinetic equation:

$$0 = t_{cyc} \frac{d\mathcal{F}}{dt} \equiv -\langle \Delta p \rangle \frac{d\mathcal{F}}{dp} - P_{esc} \mathcal{F}, \quad \mathcal{F}(p) = \int_p^\infty f(p_1) dp_1,$$

where  $f(p)$  is the distribution function.

- \* Here  $\langle \Delta p \rangle$  is the mean momentum gain per shock crossing cycle, and  $P_{esc}$  is the probability of particle loss from the cycle due to convection downstream:

$$P_{esc} \approx \frac{4u_1}{v}, \quad \langle \Delta p \rangle \approx \frac{4u_1(u_1 - u_2)p}{3u_2v}$$

for isotropic particles in a non-rel. shock.





# Non-relativistic Shocks: Canonical Index

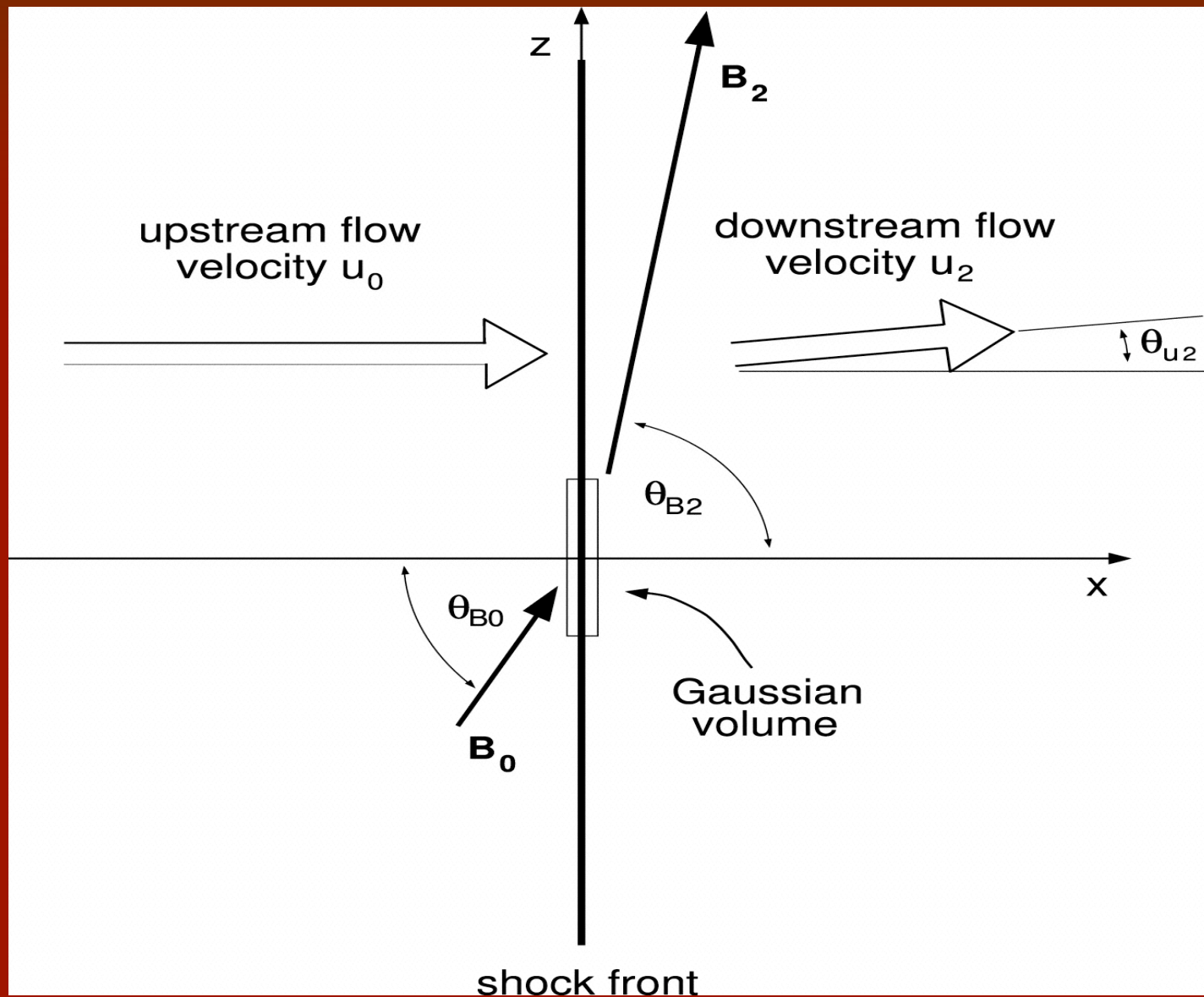
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- The kinetic equation admits power-law solutions corresponding to a *complete lack of energy scale* to the problem:

$$f(p) \propto p^{-\sigma} , \quad \sigma = 1 + \frac{p P_{\text{esc}}}{\langle \Delta p \rangle} = \frac{r + 2}{r - 1} , \quad r = \frac{u_1}{u_2} .$$

- The power-law index  $\sigma$  is independent of:
  - \* the shock obliquity  $\Theta_{\text{Bn1}}$  ,
  - \* the nature of scattering, and
  - \* the ratio  $\kappa_{\perp}/\kappa_{\parallel}$  of spatial diffusion coefficients perpendicular and parallel to the field (or equivalently  $\eta = \lambda/r_g$  ).
- Canonical index contingent upon isotropy assumption.

# Oblique Shock Geometry



# MHD Shock Conservation Equations

$$[B_x]_1^2 = 0,$$

*Mass*

$$[\rho U_x]_1^2 = 0,$$

*Momentum*

$$[\rho U_x^2 + P + B_z^2/8\pi]_1^2 = 0,$$

*Momentum*

$$[\rho U_x U_z - B_x B_z/4\pi]_1^2 = 0,$$

$$[U_z B_x - U_x B_z]_1^2 = 0,$$

*Energy*

$$\left[ \left( \frac{\gamma_r}{\gamma_r - 1} \right) P U_x + \frac{1}{2} \rho U_x U^2 + U_x B^2/4\pi - B_x (U_x B_x + U_z B_z)/4\pi \right]_1^2 = 0$$

- Rankine-Hugoniot jump conditions give downstream quantities (2) in terms of upstream quantities (1);
- They express conservation of mass, momentum, energy and magnetic fluxes (Maxwell's equations) across a **collisionless** shock.

# Rankine-Hugoniot Solutions

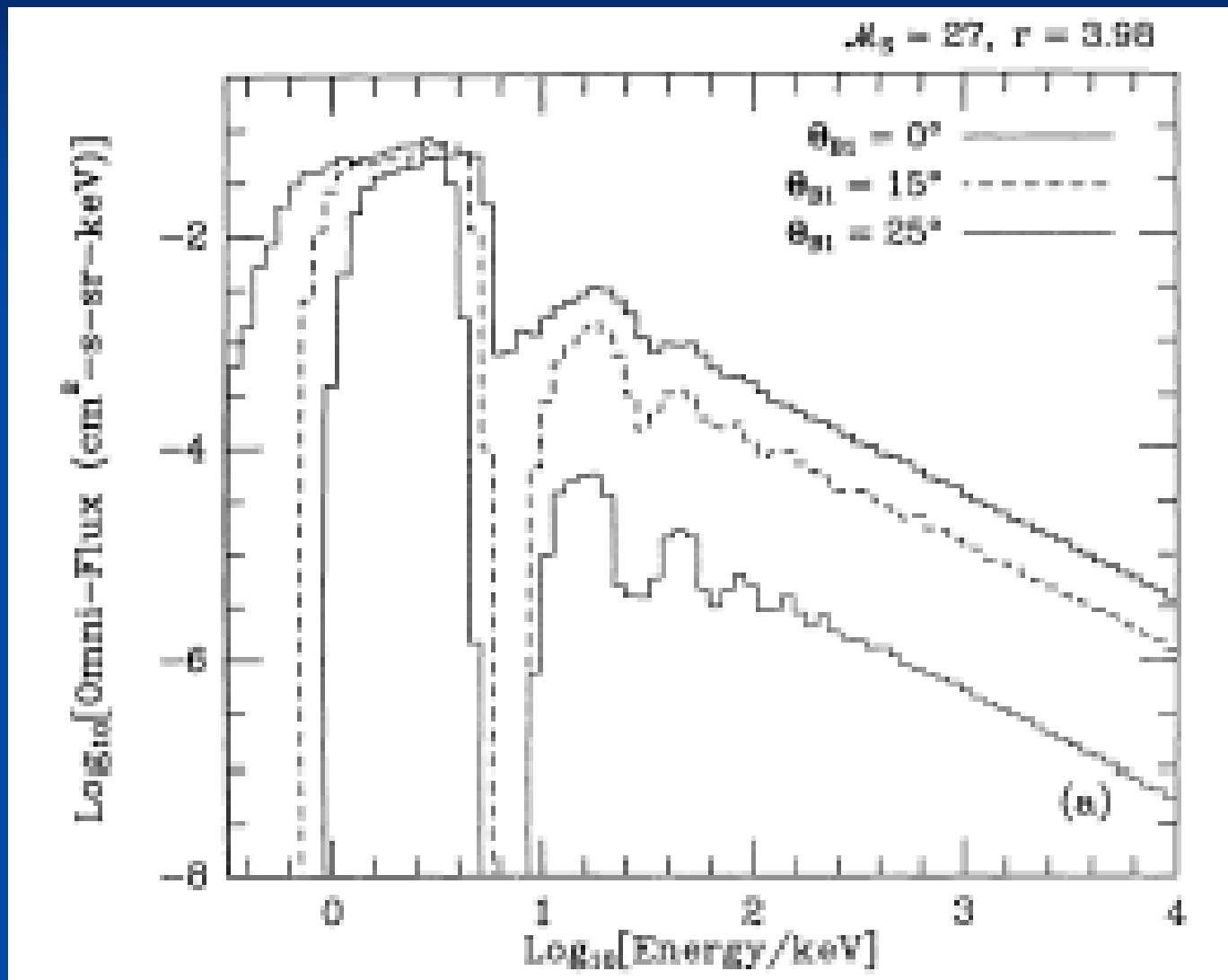
- In the hydrodynamic approximation, where the Alfvénic Mach number  $\mathcal{M}_A = u_{1x}/v_A$  (for Alfvén speed  $v_A = B_1/\sqrt{4\pi n_p m_p}$ ) far exceeds unity,

$$r \equiv \frac{u_{1x}}{u_{2x}} = \frac{\gamma_g + 1}{\gamma_g - 1 + 2/\mathcal{M}_S^2} \quad , \quad \frac{P_2}{P_1} = \frac{1 + \gamma_g(2\mathcal{M}_S^2 - 1)}{1 + \gamma_g}$$

defines the solution of the **Rankine-Hugoniot** conditions. Here,  $\mathcal{M}_S = u_{1x}/c_s = u_{1x} \sqrt{m_p/(\gamma_g k T_1)}$  is the sonic Mach number.

- So,  $r = 4$  for non-relativistic ( $\gamma_g = 5/3$ ) strong shocks with  $\mathcal{M}_S \rightarrow \infty$ ; relativistically hot gases with  $\gamma_g = 4/3$  yield  $r = 7$ .
- $r$  is called the *compression ratio*, with  $\rho_2/\rho_1 = r$ .
- When  $\mathcal{M}_A \lesssim 10$ , **B** fields contribute to the dynamics, and we have an MHD shock. Then  $r$  is reduced below the hydrodynamical value, and the field is compressed, yielding  $1 \leq B_2/B_1 \leq 4$ .

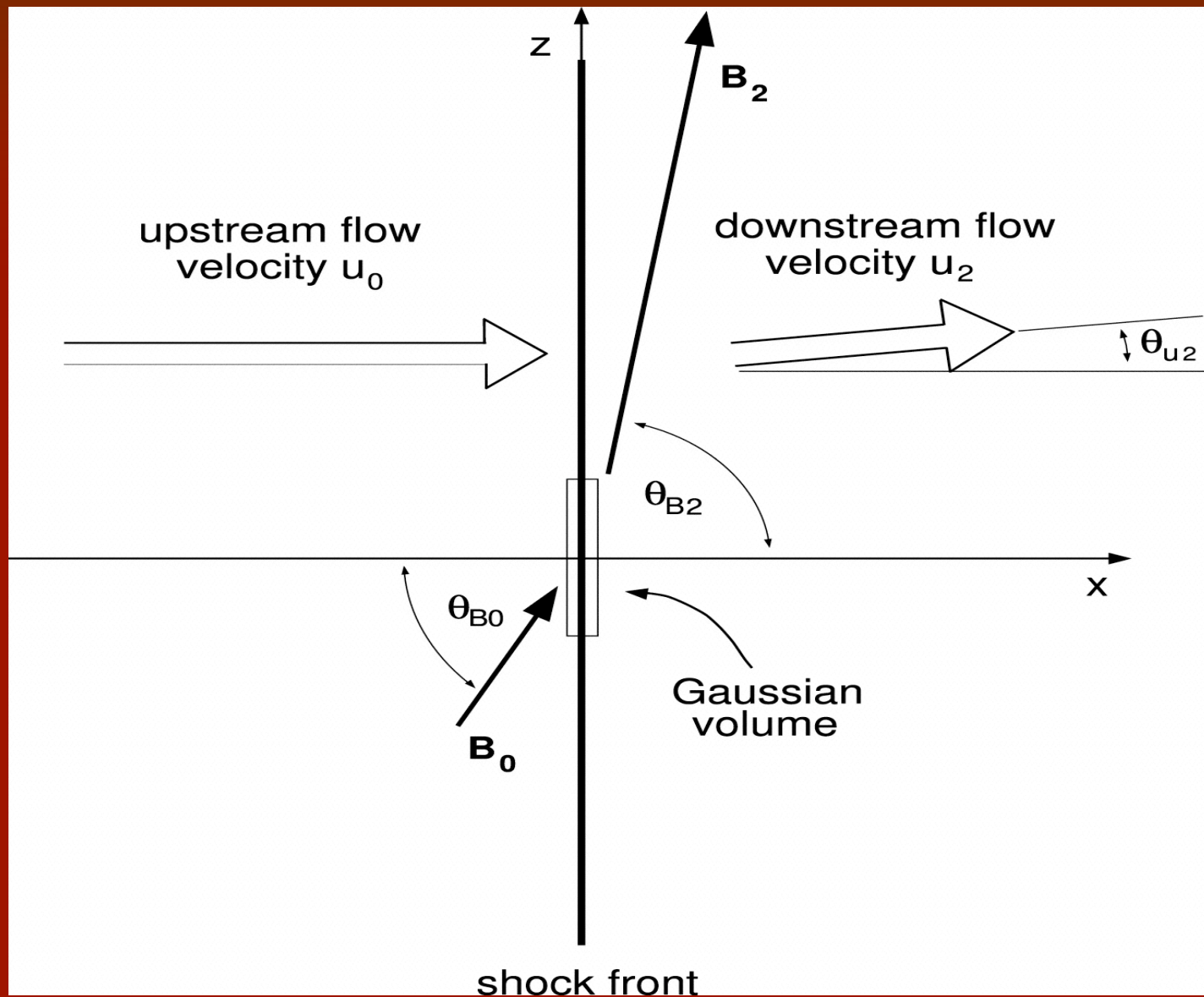
# Dependence of Distributions on Obliquity



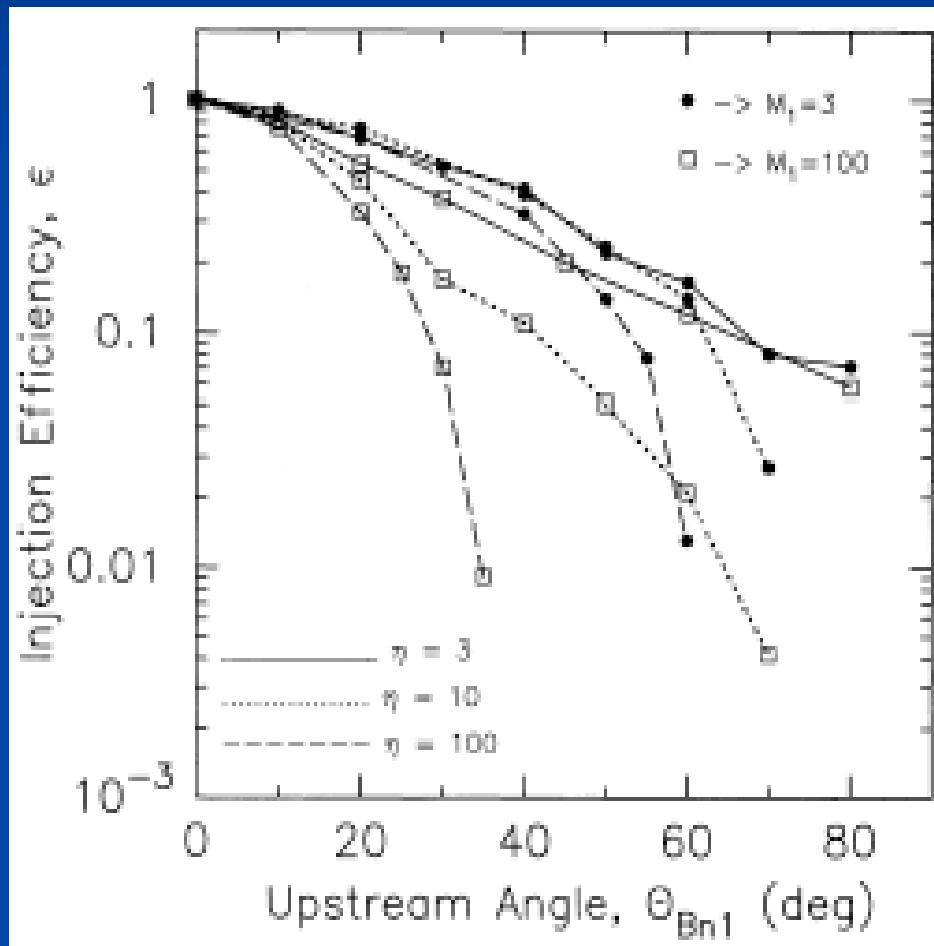
Baring, Ellison & Jones (1994)



# Oblique Shock Geometry



# Shock Acceleration: Relative Injection Efficiency



- Ellison, Baring & Jones (1995): Monte Carlo simulations;
- Cosmic ray injection efficiency *declines with obliquity, and increasing sonic Mach number* or  $\eta = \lambda / r_g$ , the mean free path to gyroradius ratio.

$$\eta = \frac{\lambda}{r_g} \quad (\eta \gtrsim 1)$$

# Acceleration Times + Maximum Energies

- For non-relativistic parallel ( $\Theta_{\text{Bn1}} = 0^\circ$ ) shocks, in the diffusion approximation (= isotropy), the acceleration time is (e.g. Forman, Jokipii & Owens 1974)

$$\tau_{acc}^{NR} = \frac{3}{u_1 - u_2} \int_{p_i}^p \left( \frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2} \right) \frac{dp'}{p'} ,$$

so that

$$\tau_{acc}^{NR} \approx \frac{0.1}{\beta_1^2} \frac{E_{\text{TeV}}}{B_{\text{Gauss}}} \text{ sec.}$$

- SNRs can almost reach the CR “knee” for  $B \sim 10\mu$  Gauss.
- AGNs can accelerate UHECRs in days if  $B \sim 100$  Gauss.
- For GRBs, the variability timescale is much shorter, thereby requiring much higher fields,  $B \sim 10^4$  Gauss.

Note: kappa is spatial diffusion coefficient

$$\kappa = \frac{1}{3} \lambda v$$

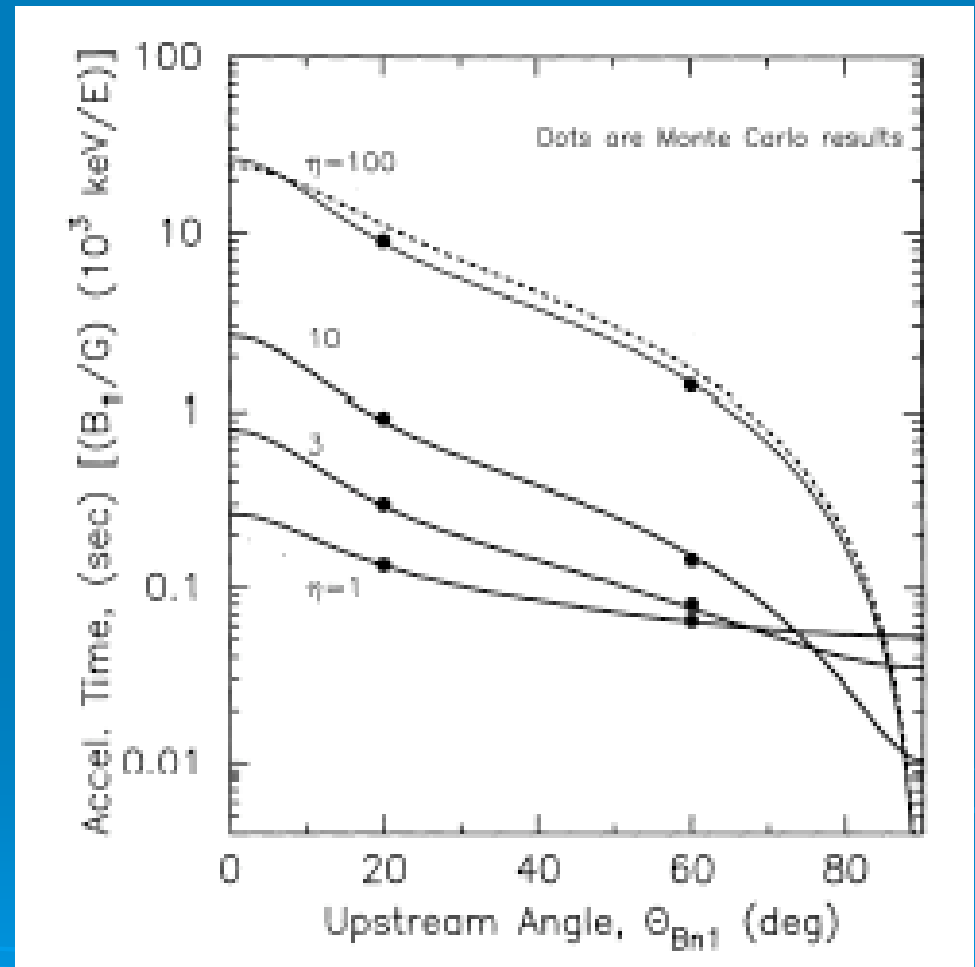
# Acceleration Times and Maximum Energies II

- In non-relativistic shocks, acceleration time scales as the inverse gyrofrequency;
- But, often the maximum energy occurs when the gyroradius equals the spatial scale of the source:
  - In SNR Sedov phase, spatial constraints more often control the maximum energy attained;
- In rotating, MHD systems (e.g. heliosphere and pulsar winds), maximum energy couples directly to the pole-to-equator potential drop;
- Shock obliquity introduces cross-field diffusion as an important influence.

$$\eta = \frac{\lambda}{r_g}$$

# Acceleration Time vs. Obliquity

- Jokipii (1987) proposed that oblique shocks would be much faster at accelerating particles;
- Origin of effect is much greater shock drift acceleration in  $\mathbf{u} \times \mathbf{B}$  drift electric field;
- Ellison et al. (1995) showed that injection was then inefficient, unless cross field diffusion was strong.



Ellison, Baring & Jones (1995)

$$\kappa_i \rightarrow \frac{\lambda v}{3} \left[ \cos^2 \Theta_{Bn1} + \frac{\sin^2 \Theta_{Bn1}}{1 + \eta^2} \right]$$

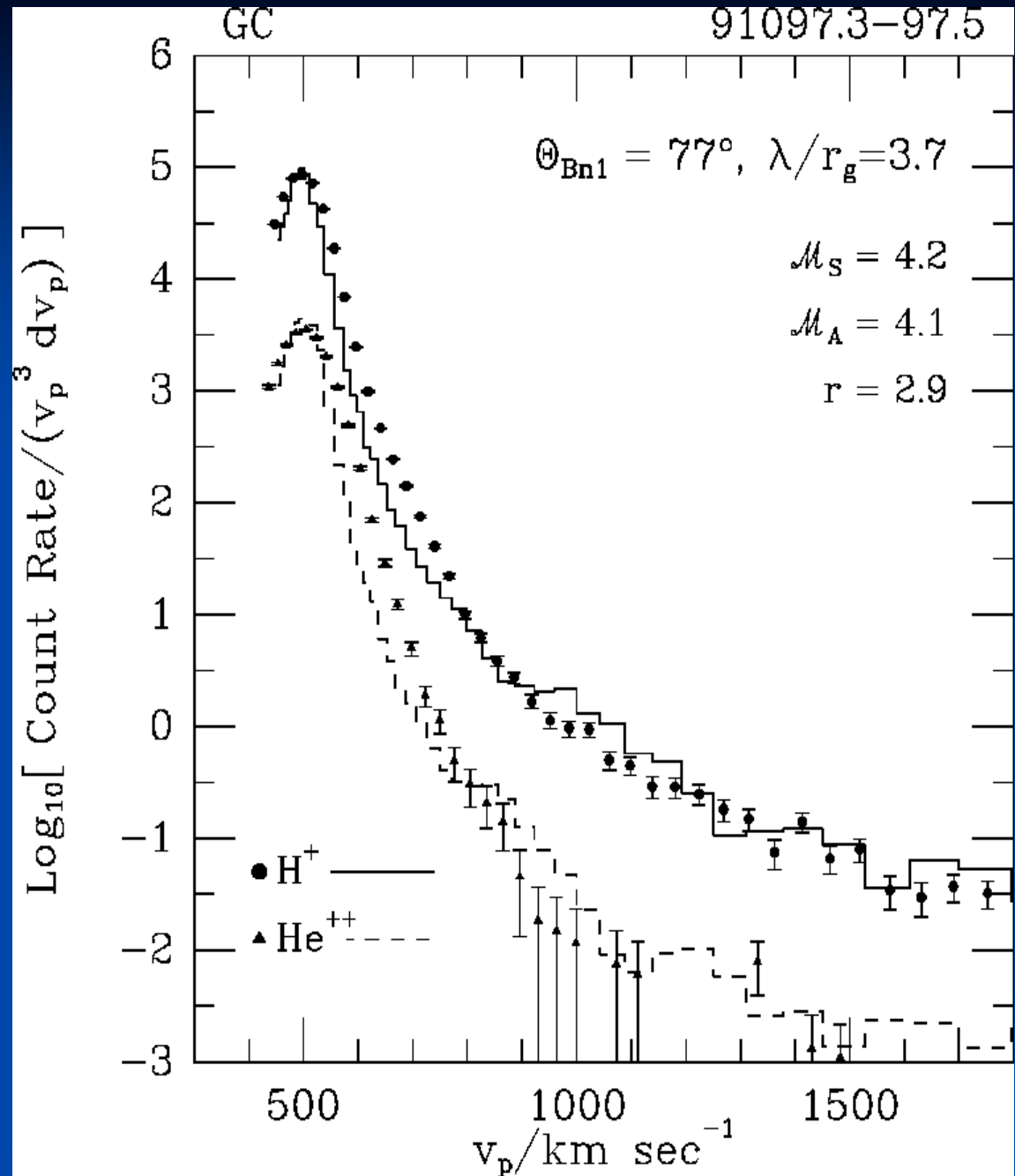
# How do we Test Acceleration Theory?

- *In situ* spacecraft measurements of particle distributions, anisotropies, relative abundances and magnetic field turbulence is the most powerful probe;
- Possible at Earth's bow shock (e.g. AMPTE), travelling interplanetary shocks (e.g. Ulysses), and the solar wind termination shock (e.g. Voyager);
  - Ions dominate the signal and therefore the diagnostics;
- Combined acceleration and radiation spectral modeling at cosmic sources such as SNRs and blazars;
- Multiwavelength approach (radio/X-ray/gamma-ray) is strongest, but is marred by indirect probe of accelerated population;
  - Electrons spawn the dominant signal, yet ions may control the acceleration characteristics.



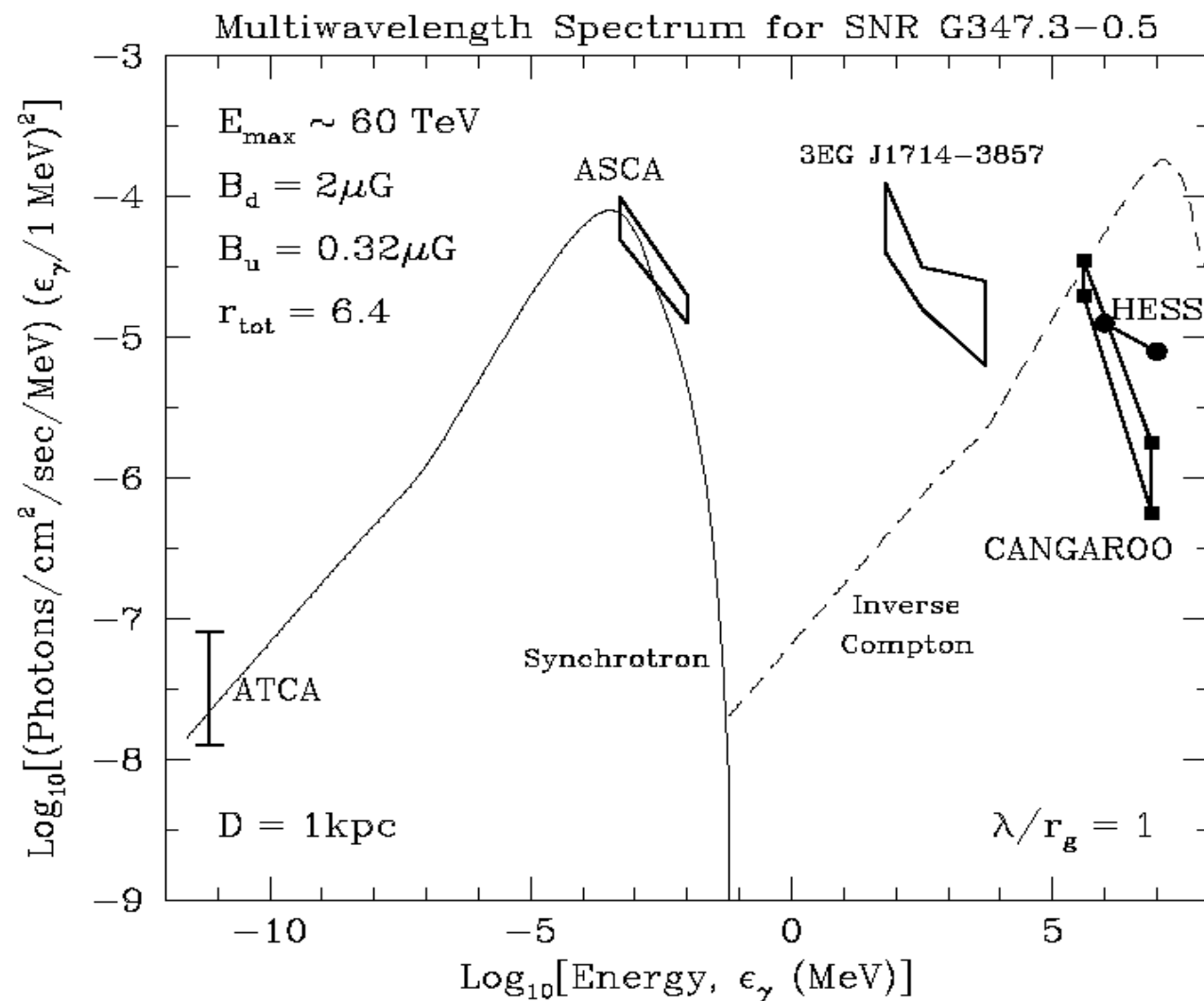
Baring, Ogilvie, Ellison  
& Forsyth 1997  
(also Kang & Jones 1997)

- Non-relativistic, low Mach number **interplanetary shocks**;
- SWICS data fit to shock of (April 7, '91) at 2.7AU;
- Shock-heated thermal ions dominate;
- **Strong cross-field diffusion needed:** same for H and alpha particles.



# Spectral Modeling of SNR Shell Emission

Baring, Ellison & Slane (2005)



# End of Lecture I: Synopsis so far

- Shock acceleration is ubiquitous in astrophysics;
- The robust prediction of canonical power-law is a (hypnotic?!) driver for its adoption in various cosmic environments;
- Magnetic field obliquity influences mostly the injection efficiency and acceleration time, not the power-law index.
- Non-relativistic, test-particle shock acceleration theory is well-researched, well understood, and confirmed by comparison with data.